RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, SEPTEMBER 2020
SECOND YEAR [BATCH 2018-21]

Date : 26/09/2020 **MATHEMATICS(Honours)**
Time : 11am – 3pm **Paper** : PAPER IV Full Marks : 50

Instructions to the students

- Write your College Roll No, Year, Subject and Paper Number on the top of the Answer Script.
- Write your Name, College Roll No, Year, Subject and Paper Number on the text box of your e-mail.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers on single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a single PDF file (Named as your College Roll No) and send it to [redacted]

Group - A (Analysis II)

Answer all the questions of this group. Maximum you can obtain 17 marks in this group.

1. Find three distinct subsets of $\mathbb{R}$ with same boundary $\{1, 2, 3\}$. Give justification. [3]
2. Prove that a complete metric space without any isolated point is uncountable. [3]
3. Let $A = \{(x, y) \in \mathbb{R}^2 : x \notin \mathbb{Q} \text{ or } y \notin \mathbb{Q}\}$. Show that A is neither open nor compact in $\mathbb{R}^2$. [4]
4. Prove that $\mathbb{Q}$ is not a G_δ subset of $\mathbb{R}$. [3]
5. Let $f : \mathbb{R} \to \mathbb{R}$ be a polynomial. Show that $f(C)$ is closed in $\mathbb{R}$ if C is closed in $\mathbb{R}$. [5]
6. Define a metric on $X = \mathbb{N} \cup \{0\}$ such that (X, d) is compact. Justify your answer. [3]

Group - B (Linear Algebra IIB)

Answer any 2 questions from 7-9 in this group. [2 x 4 = 8 marks]

7. Let V be the subspace of $R[x]$ of polynomials of degree at most 3. Equip V with the inner product $(f|g) = \int_0^1 f(t)g(t)dt$. Apply the Gram-schmidt process to the basis $\{1, x, x^2, x^3\}$. [4]

8. Show that the product of two self-adjoint operators is self-adjoint if and only if the two operators commute. [4]

9. Let T be a linear operator on a finite dimensional complex inner product space. Prove that T is normal if and only if $T = T_1 + iT_2$, where T_1 and T_2 are self adjoint operators which commute. [4]

Group - C (Differential Equation II)

Answer all the questions of this group. Maximum you can obtain 15 marks in this group.

10. Solve the differential equation [3]

$$\frac{y+z-2x}{(y-x)(z-x)}dx + \frac{z+x-2y}{(z-y)(x-y)}dy + \frac{x+y-2z}{(x-z)(y-z)}dz = 0$$

11. Factorise the operator on the left hand side of

$$[D^2 + (1-x)D - 1]y = e^x$$

and hence solve it. [4]

12. Find all the eigen-values and eigen-functions of the Sturm-Liouville problem [5]

$$(x^3y')' + \lambda xy = 0,\ y(1) = 0,\ y(e) = 0$$

13. Find $L^{-1}\left\{\frac{p^2}{p^4+4a^4}\right\}$. [3]

14. Prove that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{p}\right)$ and hence, find $L\left\{\frac{\sin at}{t}\right\}$. Does the Laplace transform of $\frac{\cos at}{t}$ exist? Justify your answer. [3]

Group - D (Applications of Calculus)

Answer all questions in this group. [2 x 5 = 10 marks]

15. Compute the signed curvature of the curve $\gamma : \mathbb{R} \to \mathbb{R}^2$ defined by $\gamma(t) = (ae^{bt}\cos t, ae^{bt}\sin t)$, where a and b are real constant, $a \neq 0$. [5]

16. Show that each of the curves

$$(x\cos\alpha - y\sin\alpha - b)^3 = c(x\sin\alpha + y\cos\alpha)^2,$$

where α is a parameter and b, c are real constants with $c \neq 0$, has a cusp and that the cusps all lie on a circle. [5]

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